(26)

$$\operatorname{tg}\left(x+\frac{\pi}{4}\right)+\operatorname{ctg}\left(x-\frac{\pi}{4}\right)=$$

$$=\frac{\operatorname{tg}x+\operatorname{tg}\frac{\pi}{4}}{1-\operatorname{tg}x\cdot\operatorname{tg}\frac{\pi}{4}}+\frac{1}{\operatorname{tg}\left(x-\frac{\pi}{4}\right)}$$

$$=\frac{\operatorname{tg}x+1}{1-\operatorname{tg}x}+\frac{1+\operatorname{tg}x\cdot\operatorname{tg}\frac{\pi}{4}}{\operatorname{tg}x-\operatorname{tg}\frac{\pi}{4}}=$$

$$=-\frac{\operatorname{tg}x+1}{-1+\operatorname{tg}x}+\frac{1+\operatorname{tg}x}{\operatorname{tg}x-1}=$$

$$=\frac{-\operatorname{tg}x-1+1+\operatorname{tg}x}{\operatorname{tg}x-1}=0$$

$$\frac{2\sin(\alpha-\beta)}{\operatorname{tg}\alpha-\operatorname{tg}\beta}=\cos(\alpha-\beta)+\cos(\alpha+\beta)$$

$$\frac{2\sin\alpha\cos\beta-2\sin\beta\cos\alpha}{\frac{\operatorname{sen}\alpha}{\cos\alpha}-\frac{\operatorname{sen}\beta}{\cos\beta}}=\cos\alpha\cos\beta+\sin\alpha\sin\beta+\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\frac{2\sin\alpha\cos\beta-2\sin\beta\cos\alpha}{\frac{\operatorname{sen}\alpha\cos\beta-\sin\beta\cos\alpha}{\cos\alpha\cos\beta}}=2\cos\alpha\cos\beta$$

$$2\left(\sin\alpha\cos\beta-\sin\beta\cos\alpha\right)\frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta-\sin\beta\cos\alpha}=2\cos\alpha\cos\beta$$

$\sin\alpha = -\frac{2}{3}$ $\quad \pi < \alpha < \frac{3}{2}\pi$ $\quad \cos\alpha = \pm\sqrt{1-\frac{4}{9}} =$

$= -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$

$\cos\beta = \frac{1}{3}$ $\quad \frac{3}{2}\pi < \beta < 2\pi$ $\quad \sin\beta = -\sqrt{1-\frac{1}{9}} =$

$$\operatorname{tg}(\alpha+\beta) = ? \quad \frac{\operatorname{sen}(\alpha+\beta)}{\cos(\alpha+\beta)} = \quad = -\frac{2\sqrt{2}}{3}$$

$$= \frac{\operatorname{sen}\alpha\cos\beta + \operatorname{sen}\beta\cos\alpha}{\cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta} =$$

$$= \frac{-\frac{2}{3}\cdot\frac{1}{3} + \left(-\frac{2\sqrt{2}}{3}\right)\cdot\left(-\frac{\sqrt{5}}{3}\right)}{-\frac{\sqrt{5}}{3}\cdot\frac{1}{3} - \left(-\frac{2}{3}\right)\left(-\frac{2\sqrt{2}}{3}\right)} =$$

$$= \frac{-\frac{2}{9} + \frac{2\sqrt{10}}{9}}{-\frac{\sqrt{5}}{9} - \frac{4\sqrt{2}}{9}} = \frac{\left(-2\sqrt{10}+2\right)\left(\sqrt{5}-4\sqrt{2}\right)}{\left(+\sqrt{5}+4\sqrt{2}\right)\left(\sqrt{5}-4\sqrt{2}\right)}$$

$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{\operatorname{sen}\alpha}{\pm\sqrt{1-\operatorname{sen}^2\alpha}}$$

57 $\sin\left(\frac{\pi}{6}+\alpha\right)$ $\operatorname{ctg}\alpha=\frac{1}{2}$.

$\pi<\alpha<\frac{3}{2}\pi$

$\sin\frac{\pi}{6}\cos\alpha+\sin\alpha\cos\frac{\pi}{6}=$

$=\frac{1}{2}\cos\alpha+\frac{\sqrt{3}}{2}\sin\alpha=$

$\operatorname{tg}\alpha=2$

$\sin\alpha=\pm\frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}}=\frac{-2}{\sqrt{5}}$

$\cos\alpha=\pm\frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}}=-\frac{1}{\sqrt{5}}$

$=\frac{1}{2}\left(-\frac{1}{\sqrt{5}}\right)+\frac{\sqrt{3}}{2}\left(-\frac{2}{\sqrt{5}}\right)=$

$=-\frac{1}{2\sqrt{5}}-\frac{2\sqrt{3}}{2\sqrt{5}}=\dots\frac{-1-2\sqrt{3}}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}$

$=\frac{-\sqrt{5}\left(1+2\sqrt{3}\right)}{10}$

8)

$$Y = \left(-\sin x + \sqrt{3}\cos x\right) + 1$$

$a = -1$ ($= \cos\alpha$) $\quad b = \sqrt{3}$ ($= \operatorname{sen}\alpha$)

$$Y = r\sin(x + \alpha)$$

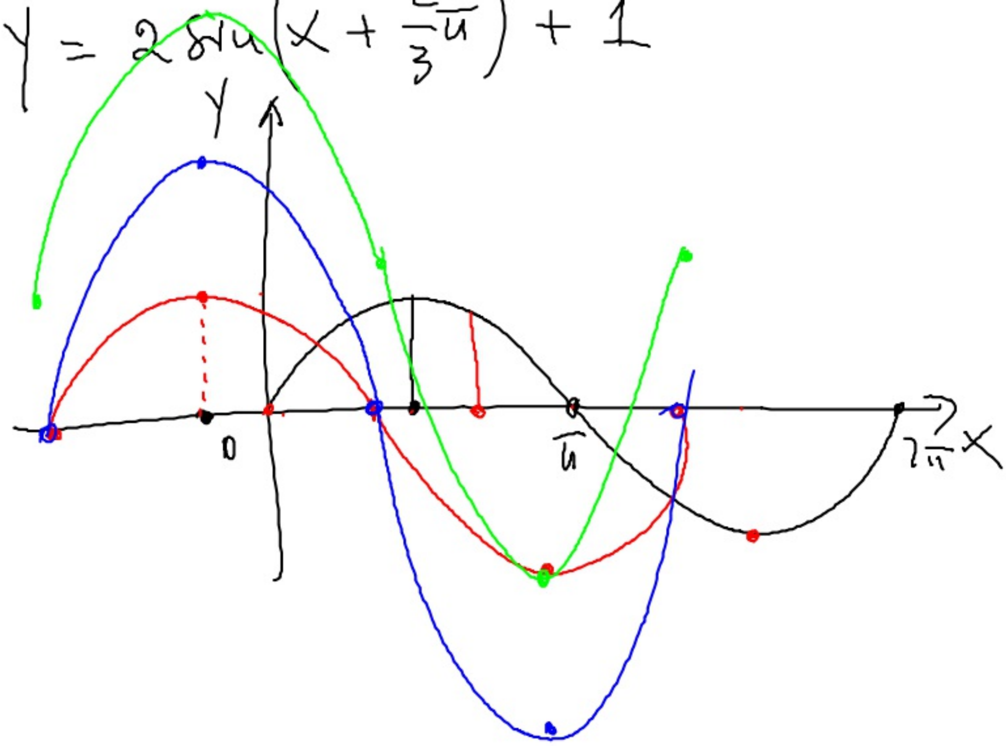
$$r = \sqrt{a^2 + b^2} = \sqrt{1 + 3} = 2$$

$$\operatorname{tg}\alpha = \frac{b}{a} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$$\boxed{\alpha_1 = \frac{2}{3}\pi}$$

$$\alpha_2 = \frac{5}{3}\pi$$

$$Y = 2\sin\left(x + \frac{2}{3}\pi\right) + 1$$



$y = \sin\left(2x + \frac{\pi}{3}\right)$ $\sin\left(\frac{3}{2}x\right)$

$2\left(x + \frac{\pi}{6}\right)$ $\frac{3}{2}x = 2\pi$

$x = \frac{2\pi \cdot 2}{3}$

$y = -5x$ $x - y - 2 = 0$ $= \frac{4}{3}\pi$

$m_1 = -5$ $m_2 = 1$

$$\operatorname{tg}\alpha = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{1+5}{1-5} = \frac{6}{-4} = -\frac{3}{2}$$

$$\operatorname{tg}\beta = \frac{3}{2}$$

FORMULE DI DUPLICAZIONE

$\cos 2\alpha = ?$

$$\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\cdot\cos\alpha - \sin\alpha\sin\alpha =$$

$$= \cos^2\alpha - \sin^2\alpha = 1-\sin^2\alpha-\sin^2\alpha =$$

$$= 1-2\sin^2\alpha = \cos^2\alpha - (1-\cos^2\alpha) =$$

$$= 2\cos^2\alpha - 1$$

$$\sin 2\alpha = \sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \sin\alpha\cos\alpha =$$

$$= 2\sin\alpha\cos\alpha$$

$$\operatorname{tg} 2\alpha = \operatorname{tg}(\alpha+\alpha) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\alpha}{1-\operatorname{tg}\alpha\cdot\operatorname{tg}\alpha} = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$$

FORMULE DI BISEZIONE

$\cos\left(\frac{\alpha}{2}\right)$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\frac{\cos 2\alpha + 1}{2} = \cos^2\alpha$

$\cos\alpha = \pm\sqrt{\frac{1+\cos 2\alpha}{2}}$

$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

$\cos 2\alpha = 1 - 2\operatorname{sen}^2\alpha$

$2\operatorname{sen}^2\alpha = 1 - \cos 2\alpha$

$\operatorname{sen}^2\alpha = \frac{1-\cos 2\alpha}{2} \Rightarrow \operatorname{sen}\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{2}}$

$\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$\operatorname{tg}\frac{\alpha}{2} = \frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{\pm\sqrt{\frac{1-\cos\alpha}{2}}}{\pm\sqrt{\frac{1+\cos\alpha}{2}}} = \pm\sqrt{\frac{\frac{1-\cos\alpha}{2}}{\frac{1+\cos\alpha}{2}}}$

$= \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

$\operatorname{tg}\frac{\alpha}{2} = \frac{\operatorname{sen}\alpha}{1+\cos\alpha} = \boxed{\frac{1-\cos\alpha}{\operatorname{sen}\alpha}}$

$\alpha \neq \pi + 2k\pi$ $\qquad$ $\alpha \neq \pi + k\pi$